

Remarks on Gribov mechanism on N=1 Supersymmetric 3D theories and the possibility of obtaining Gribov from one ABJM like Theory.

M. M. Amaral*, V. E. R. Lemes[†]

UERJ – Universidade do Estado do Rio de Janeiro

Instituto de Física – Departamento de Física Teórica

Rua So Francisco Xavier 524, 20550-013 Maracan, Rio de Janeiro, RJ, Brasil.

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Abstract

Some remarks on Gribov mechanism on N=1 Supersymmetric 3D theories are presented. The two point correlation function is analysed and the possibility of obtaining the confining Gribov regime is discussed. Also the possibility of obtaining Gribov behaviour in ABJM is presented.

*mramaciel@gmail.com

[†]vitor@dft.if.uerj.br

1 Introduction

Three-dimensional Yang-Mills (YM) theory is one important model in which it is possible to investigate nonperturbative aspects of gauge field theories such as color confinement. The theory has local degrees of freedom and the coupling constant is dimensionful. This properties indicates that this theory can be seen as an approximation for the high temperature phase of QCD with the mass gap in the role of the magnetic mass. It is important to note that in $D = 3$ it is always possible to introduce a Chern-Simons term [1, 2]. This term provides a topological mass, opening the possibility for a deconfining phase due to the appearance of a massive excitation not present in the dual superconductor picture [3, 4, 5]. Color confinement is a challenging issue that has been studied in different approaches. One of them comes from the analysis of copies of Gribov [6], known generally as Gribov problem¹, which highlights the Gribov-Zwanziger model (GZ) [8, 9, 10, 11] and its refined version (RGZ) [12]. One of the Gribov mechanism properties is that it generates propagators for gauge fields with complex poles being impossible their identification with the propagation of particle physics, which is interpreted as confinement and known generally as Gribov-Zwanziger scenario. As is widely known the presence of the Gribov problem is a general characteristic of the quantization procedure of Yang-Mills theories. In this procedure occur the existence of Gribov copies that are, in fact, a general property of all the local covariant renormalizable gauge fixing [13]. The presence of gauge copies results in zero modes of the Faddeev-Popov operator that makes the usual Faddeev-Popov construction incomplete. So the Gribov mechanism is an interesting possibility of the investigation of confinement in YM theories in three dimensions. That is, YM-CS theory with the Gribov mechanism has the possibility of a confined and a de-confined phase, depending on a fine tune between the Gribov mass gap and the usual parameters in the pure YM-CS [14]. In this case we have an interesting model with different phases.

In this context supersymmetry has been used as an important tool in the investigation of nonperturbative aspects of gauge theories. In confining behavior of supersymmetric theories, $N = 1$ for example, much has been done since the work of Seiberg with super QCD. See reference [15] and references therein that examine in detail the recent developments. And we can point out yet that in three dimensions there is currently a renewed interest in supersymmetry because it was recently found the correspondence known as ADS_4/CFT_3 [16], whose main example is the ABJM theory [16], which is closely related to CS theory in the gauge sector. This correspondence is the realization that certain theories of supergravity in higher dimensions are dual of supersymmetric quantum field theory in lower dimensions, and thus makes it possible to relate the strong coupling region of supersymmetric quantum field theory with weak coupling of the supergravity.

It is also important to note that due to the supersymmetric extension is also possible to introduce fermions in a natural way. Of course this fermions are the gauge partners and are not in the fundamental representation as quarks. In spite of that they can be useful in order to understand the behaviour of confining fermions and the possibility of relations under Gribov and supersymmetric theories.

So in this paper we investigate the Super-Yang-Mills Chern-Simons (SYM-CS) theories ($N = 1, D = 3$) with superfields formalism [17] addressing the Gribov problem and the Gribov two-point correlator

$$\mathcal{G}(p^2) = \frac{p^2}{p^4 + \gamma^4} \quad (1)$$

as well as a modified Gribov Zwanziger type correlator

$$\mathcal{G}(p^2) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + (M^2m^2 + \gamma^4)}, \quad (2)$$

and thus obtaining information on how the theory (SYM-CS) behaves in the presence of Gribov horizon and how to obtain the Gribov regime in a closest relation to the ABJM scenario.

The paper is organized as follows: in Section 2, the SYM-CS theory in superspace $D = 3, N = 1$ is presented, Landau gauge fixing is performed and the Gribov problem is analysed. In section 3 the Gribov Zwanziger local action is presented and a mathematical analysis of the value of the Gribov parameter is presented. In section 4 a possible mechanism in order to obtain a Gribov behaviour in a ABJM type theory is presented.

¹see [7] for a pedagogical review

2 Superfield approach to Gribov problem, $N = 1$, $D = 3$, SYM-CS theory.

2.1 $N = 1$, $D = 3$, Euclidean SYM-CS theory

In three-dimensional Minkowski space-time the Lorentz group is $SL(2, R)$ (instead of $SL(2, C)$) and the corresponding fundamental representation acts on a two components real spinor (Majorana) . So to formulate the superspace, we started with the introduction of spinorial coordinates θ^α (with $\alpha = 1, 2$) that are transformed under $SO(1, 2)$. In the case of Euclidean $D = 3$, the two components spinor shall be transformed under $SO(3)$ and as is well known [18, 19, 20] one can not have the usual Majorana condition. It's the same question we are in $D = 4$ [21]. In the same way we follow the approach of generalizing the concept of complex conjugation of Grassmann algebra [22]. The notations and conventions are in Appendix A.

In $D = 3$ it is possible to add an additional gauge invariant term beyond the YM, the term CS [1, 2], which is a topological mass term for the gauge field. Thus the pure supersymmetry $N = 1$ version of this action must have both terms. Let us take the Euclidean version of this superspace action of SYM-CS [23]:

$$S_{SYMCS} = S_{SYM} + S_{SCS}, \quad (3)$$

with,

$$S_{SYM} = \frac{1}{2} \int d^3x d^2\theta W^{a\alpha} W_\alpha^a, \quad (4)$$

and

$$S_{SCS} = im \int d^3x d^2\theta \left[(D^\alpha \Gamma^{a\beta})(D_\beta \Gamma_\alpha^a) + \frac{2}{3} ig f^{abc} \Gamma^{a\alpha} \Gamma^{b\beta} (D_\beta \Gamma_\alpha^c) - \frac{1}{6} g^2 f^{abc} f^{cde} \Gamma^{a\alpha} \Gamma^{b\beta} \Gamma_\alpha^d \Gamma_\beta^e \right]. \quad (5)$$

The field strength is given by:

$$W_\alpha^a = D^\beta D_\alpha \Gamma_\beta^a + ig f^{abc} \Gamma^{b\beta} D_\beta \Gamma_\alpha^c - \frac{1}{3} g^2 f^{abc} f^{cde} \Gamma^{b\beta} \Gamma_\beta^d \Gamma_\alpha^e, \quad (6)$$

and superspace derivative:

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma_\alpha^{\mu\gamma} \varepsilon_{\gamma\beta} \theta^\beta \partial_\mu. \quad (7)$$

The supermultiplet of gauge fields is given by the components of the spinor superfield, in Wess-Zumino gauge:

$$\Gamma_\alpha^a(x, \theta) = i \sigma_\alpha^{\mu\gamma} \varepsilon_{\gamma\beta} \theta^\beta A_\mu^a(x) + i \theta^2 \lambda_\alpha^a(x). \quad (8)$$

They belong to the adjoint representation of the gauge group $SU(N)$.

The classical action for SYM-CS theory, S_{SYMCS} , remains invariant under the following gauge transformation

$$\delta_\Lambda \Gamma_\alpha^a = (\nabla_\alpha \Lambda)^a, \quad (9)$$

with superspace covariant derivative:

$$\nabla_\alpha^{ab} = \delta^{ab} D_\alpha + g f^{acb} \Gamma_\alpha^c. \quad (10)$$

2.2 Gauge-fixing

In order to quantize the theory correctly we have to fix the gauge and we can do covariantly using the usual procedure of Faddeev-Popov (FP) [17, 23, 24].

In the supersymmetric Landau gauge we must implement the conditions $D^\alpha \Gamma_\alpha^a = 0$. And following the usual procedure we ended with the action of gauge fixing

$$S_{gf} = \frac{1}{4} s \left\{ \int d^3x d^2\theta (c'^a D^\alpha \Gamma_\alpha^a) \right\}, \quad (11)$$

where the Faddeev-Popov ghost fields will be scalar superfield . c'^a and c^a are the antighost and the ghost respectively. And s is the BRST nilpotent operator ($s^2 = 0$).

The total action $S = S_{SYMCS} + S_{gf}$ is invariant under the BRST transformations [23]:

$$\begin{aligned} s\Gamma_\alpha^a &= (\nabla_\alpha c)^a \\ sc^a &= -\frac{i}{2}gf_{abc}c^bc^c \\ sc'^a &= b^a \\ sb^a &= 0, \end{aligned} \tag{12}$$

with s carrying ghost number 1.

The ghost part of gauge fixing action becomes:

$$S_{FP} = -\frac{1}{4} \int d^3x d^2\theta (c' D^\alpha \nabla_\alpha^{ab} c^b), \tag{13}$$

with superspace covariant derivative given by (10):

In order to calculate the propagator for the gauge superfield Γ_α , we need only the bilinear of S . So, for the bilinear part, we have:

$$\begin{aligned} S_{SYM2} &= \frac{1}{2} \int d^3x d^2\theta (D^\beta D^\alpha \Gamma_\beta^a) (D^\gamma D_\alpha \Gamma_\gamma^a) \\ &= \frac{1}{2} \int d^3x d^2\theta \Gamma_\beta^a D^\alpha D^\beta D^\gamma D_\alpha \Gamma_\gamma^a, \end{aligned}$$

and using (84, 92, 95, 96):

$$S_{SYM2} = \int d^3x d^2\theta \Gamma_\beta^a D^2 D^\gamma D^\beta \Gamma_\gamma^a,$$

and for Chern-Simons:

$$\begin{aligned} S_{SCS2} &= im \int d^3x d^2\theta [(D^\alpha \Gamma^{a\beta}) (D_\beta \Gamma_\alpha^a)] \\ &= im \int d^3x d^2\theta [\Gamma_\beta^a D^\gamma D^\beta \Gamma_\gamma^a]. \end{aligned}$$

So, with $A = D^2 + im$:

$$(AD^\gamma D^\beta + \frac{1}{\xi} D^\beta D^\gamma) (a_1 D_\beta D_\lambda + a_2 D_\lambda D_\beta) = \delta_\lambda^\gamma,$$

where ξ is a gauge parameter to be set to zero after having evaluated the gauge propagator. Using (92, 96, 84, 95) and (93), (in Landau gauge $\xi \rightarrow 0$, $a_2 = 0$) we get the inverse

$$\frac{1}{2A\partial^2} D_\beta D_\lambda,$$

so the massive gauge propagator for SYM-CS:

$$\langle \Gamma_\alpha^a(1) \Gamma_\beta^b(2) \rangle = \frac{\delta^{ab}}{\partial^2(-\partial^2 + m^2)} (D^2 - im) D_\beta D_\alpha \delta^2(\theta_1 - \theta_2) \delta^3(x_1 - x_2). \tag{14}$$

2.3 Gribov problem

In the previous section we showed generally the quantization of SYM-CS theory by Faddeev-Popov method. But one should be careful with this procedure. In the usual YM theory, although the gauge be fixed by the Faddeev-Popov method, Gribov showed in [6] that there are still field configurations obeying the Landau gauge linked by gauge transformations, *i.e.* there are still equivalent configurations, or copies, being taken into account into the Feynman path integral. In other words, the gauge is not completely fixed and the remaining ambiguity is allowed due to the existence of normalizable zero-modes of the Faddeev-Popov operator,

$$\mathcal{M}^{ab} = -\partial_\mu D_\mu^{ab}. \tag{15}$$

So we should investigate how this problem appears in the SYM-CS case and its consequences. Before we give a brief review of the YM case in d dimensions.

2.3.1 The Gribov problem in YM theories in d dimensions

To address the problem of Gribov copies (eliminate these copies) Gribov showed that the domain of integration of the functional integral should be restricted to a certain region Ω , the so-called Gribov region, that is defined as the set of field configurations performing the Landau gauge condition, for which the Faddeev-Popov operator is strictly positive, namely

$$\Omega := \{ A_\mu^a \mid \partial_\mu A_\mu^a = 0, \mathcal{M}^{ab}(A) > 0 \}. \quad (16)$$

Its boundary, $\partial\Omega$, where the first vanishing eigenvalue of the Faddeev-Popov operator shows up, is known as the Gribov horizon.

As in the region Ω the Faddeev-Popov operator is positive than its inverse must diverge when approaching the horizon, due to the existence of a zero mode. So the restriction to the first Gribov region is implemented requiring that

$$G(p^2, A) = \frac{\delta^{ab}}{N^2 - 1} \langle p | (-\partial_\mu D_\mu^{ab})^{-1} | p \rangle, \quad (17)$$

which is the normalized trace of the ghost connected two point function in momentum space, has no pole for a given non vanishing value of the momentum p , except for the singularity at $p = 0$, corresponding to the first Gribov horizon. At $p \approx 0$ one can write

$$G(p^2, A) \approx \frac{1}{p^2} \frac{1}{1 - \sigma(p^2, A)}, \quad (18)$$

$$\sigma(p^2, A) = \frac{N}{N^2 - 1} \frac{1}{p^2} \int \frac{d^d q}{(2\pi)^d} \frac{(p - q)_\mu p_\nu}{(p - q)^2} A_\mu^a(-q) A_\nu^a(q). \quad (19)$$

From the above expression (19), it follows that the no-pole condition at finite non vanishing p is

$$\sigma(p^2, A) < 1. \quad (20)$$

As $\sigma(p^2, A)$ decreases as p^2 increases one can also take

$$\sigma(0, A) = \frac{1}{4} \frac{N}{N^2 - 1} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2} (A_\mu^a(-q) A_\mu^a(q)) \leq 1. \quad (21)$$

In order to perform the restriction to the Gribov region into the partition function, \mathcal{Z} , the final step is to introduce the no-pole condition with the help of a Heaviside function:

$$\mathcal{Z} = \int \mathcal{D}A \delta(\partial A) \theta(1 - \sigma(0, A)) \exp^{-S_{\text{YM}}}. \quad (22)$$

where the Euclidean $SU(N)$ Yang-Mills action in the Landau gauge (S_{YM}) in d dimensions is given by:

$$S_{\text{YM}} = \int d^d x \left(\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + i b^a \partial_\mu A_\mu^a + \bar{c}^a \partial_\mu D_\mu^{ab} c^b \right). \quad (23)$$

Note that the only allowed singularity at (18) is at $p^2 = 0$, whose meaning is that of approaching the horizon, where $G(p^2, A)$ is singular due to the appearance of zero modes of the Faddeev-Popov operator. Thus we have to take [6]:

$$\sigma(0, A) = 1. \quad (24)$$

And thus the Gribov parameter γ is fixed by the gap equation, which is, in the case of $d = 3$ e.g.,

$$\frac{2Ng^2}{3} \int \frac{d^3 q}{(2\pi)^3} \frac{1}{q^4 + \gamma^4} = 1. \quad (25)$$

It is clear that the Gribov approach is only the first step in order to consistently treat the problem of zero modes and the Gribov copies in a gauge fixed Yang-Mills theory. The second step is the GZ theory [10, 11], which

consists in a renormalizable and local way to implement the restriction to the first Gribov region. In fact, Zwanziger observed that the restriction could be implemented by adding the following term in the action (23):

$$S_{GZ} = S_{YM} + \gamma^4 H(A), \quad (26)$$

where, $H(A)$ is the so-called horizon function,

$$H(A) = g^2 \int d^d x d^d y f^{abc} A_\mu^b(x) [\mathcal{M}^{-1}]^{ad}(x, y) f^{dec} A_\mu^e \quad (27)$$

where \mathcal{M}^{-1} stands for the inverse of the Faddeev-Popov operator. In the Zwanziger approach, the parameter γ is fixed by the equation

$$\langle H(A) \rangle = dV(N^2 - 1), \quad (28)$$

where V is the Euclidean space volume.

It is clear that the horizon function is nonlocal, but it can be localized with the help of a suitable set of auxiliary fields. In order to ensure that those extra fields do not introduce extra degrees of freedom they are introduced in the form of a BRST quartet

$$\begin{aligned} s\bar{\omega}_\mu^{ab} &= \bar{\varphi}_\mu^{ab}, & s\bar{\varphi}_\mu^{ab} &= 0, \\ s\varphi_\mu^{ab} &= \omega_\mu^{ab}, & s\omega_\mu^{ab} &= 0, \end{aligned} \quad (29)$$

where $(\bar{\varphi}, \varphi)$ are a pair of complex commuting fields, while $(\bar{\omega}, \omega)$ are anti-commuting ones. Now, the local version of the GZ action is then given by:

$$S_{GZ}^{local} = S_{YM} + s \int d^4 x \left[\bar{\omega}_\mu^{ac} \mathcal{M}^{ab} \varphi_\mu^{bc} \right] + \int d^4 x \left[\gamma^2 g f^{abc} A_\mu^a (\varphi_\mu^{bc} - \bar{\varphi}_\mu^{bc}) + dV(N^2 - 1)\gamma^4 \right]. \quad (30)$$

The last term is a vacuum term permitted by power counting and required to obtain the gap equation (28) by the condition that the vacuum energy, \mathcal{E} , be independent of γ^2 , *i.e.*,

$$-\frac{\partial \mathcal{E}}{\partial \gamma^2} = 0, \quad (31)$$

where the $-\mathcal{E}$, is defined by

$$e^{-\mathcal{E}} = \int [d\Phi] e^{-S_{GZ}}, \quad (32)$$

and $[d\Phi]$ stands for the integration over all the fields.

2.3.2 SYM-SC and Gribov problem

As discussed in the introduction, this problem of Gribov copies is a general property of all the local covariant renormalizable gauge fixing [13]. And we explicitly have shown that Gribov problem also exists in SYM in Landau gauge [21] in $D = 4$. So let's investigate how this problem appears in the gauge fixing procedure of SYM-CS theory.

First we note that in the Landau-gauge the gauge condition is not ideal. In fact if we consider two equivalents superfield, Γ_α^a and $\Gamma_\alpha^{a'}$, connected by a gauge transformation (9), if both satisfy the same condition of the Landau gauge, $D^\alpha \Gamma_\alpha^a = 0$ and $D^\alpha \Gamma_\alpha^{a'} = 0$, we have

$$D^\alpha (\nabla_\alpha \Lambda)^a = 0. \quad (33)$$

Therefore, the existence of infinitesimal copies, even after FP quantization is related to the presence of the zero modes of the operator above. This operator is the same that results in FP action (13) and so, if he has zero mode issue there are problem with functional generator

$$Z = \int D\Gamma e^{-S_{SYMSC}(\Gamma)}. \quad (34)$$

This suggests that we should further restrict the functional integration to a region free of zero modes, and consecutively free of gauge superfields copies. To do this we would like to study the operator (33) in terms of the eigenvalues

and eigenvectors equation, which is not immediate since the equation $D^\alpha(\nabla_\alpha\Lambda)^a = \lambda\Lambda$ is not an eigenvalue equation, which can be seen in components. It relates the component at $\theta = 0$ with to component θ^2 of superfield Λ . This indicates that the correct operator, where one can study the zero modes problem, and thus define the Gribov problem is:

$$\mathcal{O}^{ab} = D^2 D^\alpha \nabla_\alpha^{ab}. \quad (35)$$

This operator is the correct generalization of the FP operator² since the action (13) has an integral in θ^2 . So to see the zero mode problem we take the eigenvalues equation

$$D^2 D^\alpha \nabla_\alpha^{ab} \Lambda = \lambda \Lambda \quad (36)$$

So for configurations close to the vacuum $\Gamma_\alpha = 0$ and using (96)

$$-\partial^2 \Lambda = \lambda \Lambda, \quad (37)$$

which has only positive eigenvalues $\lambda = p^2 > 0$, since the operator in question is Hermitian. However as we go considering larger amplitudes than the vacuum, ie, Γ_α sufficiently large, this can not be guaranteed and may be displayed negative eigenvalues. So we can consider the restriction of functional integration for the region free of zero modes of this operator, which will be the generalization of the Gribov region (16)

$$\Omega := \{ \Gamma_\alpha^b \mid D^\alpha \Gamma_\alpha^b = 0, \mathcal{O}^{ab}(\Gamma_\alpha) > 0 \}. \quad (38)$$

In order to implement this restriction then we consider the GZ approach [8, 9, 10, 11] where is included in the functional integral the inverse of this operator (horizon function) in order to compensate the problem, this is formally

$$H(\Gamma_\alpha) = \gamma^4 \int d^2\theta \int d^3x d^3y f^{abc} \Gamma_\alpha^b(x) \left[\frac{\varepsilon^{\alpha\beta}}{D^2 D^\alpha \nabla_\alpha} \right]^{ad} (x, y) f^{dec} \Gamma_\beta^e(y). \quad (39)$$

In the next section we will put this horizon function in your local form and study some implications showing its consistency. The calculation of the field configurations that corresponds to the solutions of the Gribov condition is an extensive work and requires further studies.

3 Gribov-Zwanziger local action on superspace

To localize the horizon function (39) we observe that we can get rid of the inverse of the FP operator with the aid of standard formula for Gaussian integration and the introduction of a pair of spinor superfields of bosonic character and other fermionic, ending with

$$S_{aux} = tr \int d^3x d^2\theta \left[-(w'_\gamma \varepsilon^{\gamma\beta} D^2 D^\alpha \nabla_\alpha w_\beta) + (u'_\gamma \varepsilon^{\gamma\beta} D^2 D^\alpha \nabla_\alpha u_\beta) + 2\gamma^2 \Gamma_\gamma \varepsilon^{\gamma\beta} (u'_\beta - u_\beta) \right], \quad (40)$$

There exists a freedom of redefinition of fields and we can perform a linear shift on the field w_β in order to write the first two terms as a BRST variation. This can be seen best by introducing of the auxiliary spinor superfields in the form of one quartet of BRST:

$$\begin{aligned} sw'_\alpha &= u'_\alpha, \quad su_\alpha = w_\alpha \\ su'_\alpha &= 0, \quad sw_\alpha = 0, \end{aligned} \quad (41)$$

At this point is important for our construction to show the canonical dimension and ghost number of all fields and operators which are in Table 1.

Thus, we ended up with the following proposal to the super GZ action:

$$S_{aux} = tr \int d^3x d^2\theta \left[s(w'_\gamma \varepsilon^{\gamma\beta} D^2 D^\alpha \nabla_\alpha u_\beta) + 2\gamma^2 \Gamma_\gamma \varepsilon^{\gamma\beta} (u'_\beta - u_\beta) \right], \quad (42)$$

²We note that the usual Faddeev-Popov operator is the component θ^2 of the supersymmetric FP operator that appears (13).

fields and operators	θ^α	D_α	Γ_α	c'	c	b	w'_α	w_α	u'_α	u_α	g
Canonical dimension	$-\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	0	0	0	0	$\frac{1}{2}$
Ghost number	0	0	0	-1	1	0	-1	1	0	0	0

Table 1: Quantum numbers of fields and operators.

where γ^2 is a mass parameter, which should be determined by the theory, shown below, must be nonzero. And also include a vacuum term. The total action is:

$$S_{SGZ} = S_{SYMCS} + S_{gf} + S_{aux}. \quad (43)$$

With GZ action generalization at our disposal we can now calculate the propagators and ensure they have the expected behavior that occurs in confining YM theories and analyze other features it adds to SYM.

Before starting the calculation of the super propagator it is important to emphasize here that the $N = 1$ super Yang-Mills is a renormalizable action in $D = 4$ [30] and the Gribov procedure for $N = 1$, $D = 4$ super Yang-Mills was implemented in terms of component fields in [31] and directly in superspace in [21]. Also is well know that Yang-Mills-Chern-Simons is a renormalizable gauge theory and the BRST breaking in the Gribov procedure is a soft breaking that does not spoil the renormalizability, explicitly proven in $D = 4$ [32, 33, 34]. Due to power counting, the Gribov procedure must also be renormalizable in $D = 3$.

3.1 The super gauge propagator

First we calculate the propagator for gauge superfield Γ_α . To calculate the gauge propagator we need only the bilinear of S like to calculate (14). Thus, for S_{SGZ} , we have (with 96):

$$S_{SGZ2} = tr \int d^3x d^2\theta (-u'_\gamma \varepsilon^{\gamma\beta} \partial^2 u_\beta + w'_\gamma \varepsilon^{\gamma\beta} \partial^2 w_\beta + 2\gamma^2 \Gamma_\gamma \varepsilon^{\gamma\beta} u'_\beta - 2\gamma^2 \Gamma_\gamma \varepsilon^{\gamma\beta} u_\beta). \quad (44)$$

With give one contribution to bilinear term

$$S_{SGZ2} = tr \int d^3x d^2\theta \Gamma_\gamma \frac{2\gamma^4}{\partial^2} \varepsilon^{\gamma\beta} \Gamma_\beta. \quad (45)$$

Similar to SYM-CS propagator calculus, with $A = D^2 + im$ and $B = \frac{2\gamma^4}{\partial^2}$:

$$(AD^\gamma D^\beta + \frac{1}{\xi} D^\beta D^\gamma + B\varepsilon^{\gamma\beta})(a_1 D_\beta D_\lambda + a_2 D_\lambda D_\beta) = \delta_\lambda^\gamma, \quad (46)$$

and we get the inverse:

$$\frac{1}{2A\partial^2 + BD^2} D_\beta D_\lambda, \quad (47)$$

or

$$-\frac{1}{2} \left[\frac{(\partial^4 + \gamma^4) + im\partial^2 D^2}{(\partial^4 + \gamma^4)^2 - m^2(\partial^2)^3} \right] D^2 D_\beta D_\lambda. \quad (48)$$

So the gauge propagator for SYM-CS-GZ:

$$\langle \Gamma_\alpha^a(1) \Gamma_\beta^b(2) \rangle = \frac{1}{2} \delta^{ab} \left[\frac{(\partial^4 + \gamma^4) + im\partial^2 D^2}{-(\partial^4 + \gamma^4)^2 + m^2(\partial^2)^3} \right] D^2 D_\beta D_\alpha \delta^2(\theta_1 - \theta_2) \delta^3(x_1 - x_2). \quad (49)$$

To see how the introduction of S_{SGZ} brings light on confinement of both bosons as fermions and to compare with literature, we shall observe the propagators in field components.

Taking components from (8) we can project the propagator for the gauge field A_μ :

$$\langle A_\mu^a(x_1) A_\nu^b(x_2) \rangle = \delta^{ab} \left[\frac{(\partial^4 + \gamma^4)(-\partial^2)}{(\partial^4 + \gamma^4)^2 - m^2(\partial^2)^3} \right] (\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2} - \frac{im\partial^2 \varepsilon_{\mu\nu\sigma} \partial_\sigma}{(\partial^4 + \gamma^4)}) \delta^3(x_1 - x_2), \quad (50)$$

and gaugino λ^α :

$$\langle \lambda_\alpha^a(x_1) \lambda_\beta^b(x_2) \rangle = \frac{1}{4} \delta^{ab} \left[\frac{(\partial^4 + \gamma^4)}{(\partial^4 + \gamma^4)^2 - m^2(\partial^2)^3} \right] (\partial^2 \partial_{\beta\alpha} - \frac{im(\partial^2)^3 \varepsilon_{\beta\alpha}}{(\partial^4 + \gamma^4)}) \delta^3(x_1 - x_2). \quad (51)$$

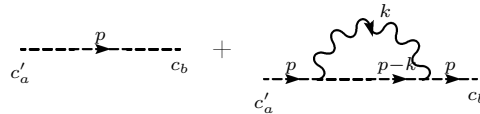
And we found that both show behavior in limit $m = 0$ as occurs for gauge field in non-supersymmetric theories (1). However as we'll see in the next section the Gribov parameter γ can be determined as a function of coupling constant g such that these propagators will be function of the two parameters, g and m (in the case $m \neq 0$), and so it is possible the study of phases involving this theory. The phases for this type of propagator was studied in [14] through the analysis of the poles in this propagator. It is important to note here that due to a Gribov type propagator also for the fermionic partners we have a fermionic condensate defined by $\lim_{x_1 \rightarrow x_2} Tr \langle \lambda_\alpha^a(x_1) \lambda_\beta^b(x_2) \rangle$. This condensate is responsible for maintaining the energy equal to zero and compensate the gauge condensate that usually appears in GZ. In simple terms, the existence of the fermion condensate ensures that the supersymmetry is preserved in spite of breaking the BRST symmetry as usually happens in GZ.

3.2 Ghost propagators and γ parameter

Since the action (43) only makes sense if the γ parameter is nonzero, we will now explicitly show that it is not independent in this theory. Its determination is closely linked to the restriction of the functional integration to the first Gribov region, which we will discuss some details here.

First, it is noteworthy that in the literature dealing with the Gribov problem in YM theories there are recent consensus on the scenario of dominance of configurations on the Gribov horizon on the Landau gauge [35], so that the restriction to the first Gribov region is, in practice, to take the configurations on the horizon, ie where occur the zeros modes of the FP operator. Second, and as we have pointed out in the introduction of super GZ, calculate the propagator of the ghosts is to take the inverse of these operators. So we focus on these calculus to one loop order to establish the one loop gap equation Gribov style.

In order to characterize the integration in the first Gribov region it is important to remember that the two point ghost function is essentially the inverse of the Faddeev-Popov operator and the zero eigenvalue of the Gribov equation corresponds to a exactly to the Gribov frontier. In these sense the two point ghost function goes to infinity at the Gribov frontier. These condition is the most simpler way to obtain the gap equation for γ . These procedure is explained in details in [6] and is easily extended to the $N = 1$ supersymmetric case [21]. First we need to calculate the two point ghost function. Using perturbation theory these is at first order of the form:



Where the line between c'_a and c_b corresponds to the zero order super ghost propagators $\mathcal{G}_{c'_c}^{ab0} = -\Delta_{c'_c}^c(1, 2)$. After a straightforward calculation:

$$\mathcal{G}_{c'_c}^{ab0} = \frac{-2}{p^2} \delta^{ab} D^2 \delta^2(\theta_1 - \theta_2). \quad (52)$$

And we can define in momentum space, the one loop corrected ghost propagator as

$$\mathcal{G}_{c'_c}^{ab} = (\mathcal{G}_{c'_c}^{ab0} + \mathcal{G}_{c'_c}^{ab1}), \quad (53)$$

according to diagram above. With $\mathcal{G}_{c'_c}^{ab0}$ given from (52).

Using the Feynman rules and D algebra from [17, 36, 37], (and due $f^{acd} f^{bcd} = N \delta^{ab}$):

$$\begin{aligned} \mathcal{G}_{c'_c}^{ab1} = & (2\pi)^3 g^2 N \delta^{ab} \int d^2\theta_3 \int d^2\theta_4 \frac{1}{p^2} D_1^2 \delta^2(\theta_1 - \theta_3) \\ & D_3^\alpha \left\{ \int \frac{d^3k}{(2\pi)^3} \frac{(k^4 + \gamma^4)}{(k^4 + \gamma^4)^2 + m^2 k^6} \frac{1}{(p-k)^2} \left(1 + \frac{imk^2 D_3^2}{(k^4 + \gamma^4)} \right) D_3^2 D_\beta D_\alpha \delta^2(\theta_3 - \theta_4) D_3^2 (p-k) \delta^2(\theta_3 - \theta_4) \right\} \overleftarrow{D}_4^\beta \\ & \frac{1}{p^2} D_4^2 \delta^2(\theta_4 - \theta_2). \end{aligned} \quad (54)$$

And after delta functions and D derivatives manipulations [37], we have:

$$\mathcal{G}_{c'c}^{ab1} = -4(2\pi)^3 g^2 N \delta^{ab} \frac{1}{p^2} D_1^2 \delta^2(\theta_1 - \theta_2) \int \frac{d^3 k}{(2\pi)^3} \left(\frac{(k^4 + \gamma^4)}{(k^4 + \gamma^4)^2 + m^2 k^6} \right) \frac{k^2}{(p-k)^2}. \quad (55)$$

Next we define:

$$\sigma(\gamma^2, p^2, m^2) = 2(2\pi)^3 g^2 N \int \frac{d^3 k}{(2\pi)^3} \left(\frac{(k^4 + \gamma^4)}{(k^4 + \gamma^4)^2 + m^2 k^6} \right) \frac{k^2}{(p-k)^2}. \quad (56)$$

Therefore, from (53):

$$\mathcal{G}_{c'c}^{ab} = -2\delta^{ab} \frac{1}{p^2} D_1^2 \delta^2(\theta_1 - \theta_2) (1 + \sigma). \quad (57)$$

Re-summing the one-particle irreducible diagrams gives:

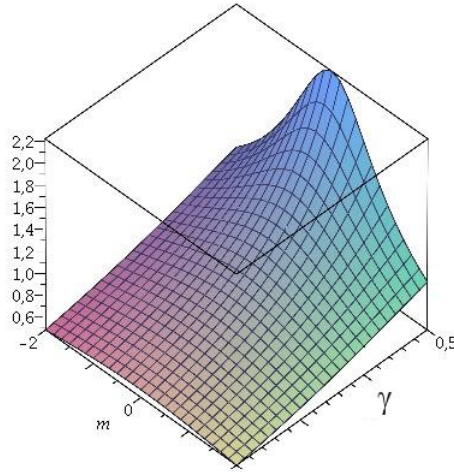
$$\mathcal{G}_{c'c}^{ab} = -2\delta^{ab} \frac{1}{p^2} D_1^2 \delta^2(\theta_1 - \theta_2) \frac{1}{(1 - \sigma)}. \quad (58)$$

Now, as we are interested in the low momentum behavior we analyze the behavior of $(1 - \sigma)$ we get:

$$\sigma(\gamma^2, 0, m^2) = 2(2\pi)^3 g^2 N \int \frac{d^3 k}{(2\pi)^3} \left(\frac{(k^4 + \gamma^4)}{(k^4 + \gamma^4)^2 + m^2 k^6} \right). \quad (59)$$

According to the above discussion of the scenario of dominance of configurations on the Gribov horizon, ie the ghost propagator (the inverse of FP operator) going to infinity, $(1 - \sigma) = 0$, we have to get the greatest value of the above integral witch is with $m = 0$ as we can see in the integral graph shown in Figure 1:

Figure 1: Graph of integral $\sigma(\gamma, m)$ (59)



At this point is important to remember that also in the Gribov procedure applied to non supersymmetric Yang-Mills it is necessary to analyse the greatest value of the integral σ . In the non supersymmetric Yang-Mills in $D = 4$ we also take the integral at zero external momenta, which corresponds to the greatest value of the integral σ . It is easy to note in Figure 1 that the maximum value for the integral will be obtained at $m = 0$, as it can be seen in the graduated axis that corresponds to σ . So we should take

$$\sigma(\gamma^2, 0, 0) = 2(2\pi)^3 g^2 N \int \frac{d^3 k}{(2\pi)^3} \left(\frac{1}{k^4 + \gamma^4} \right). \quad (60)$$

And so we are able to define the one loop gap equation :

$$2(2\pi)^3 g^2 N \int \frac{d^3 k}{(2\pi)^3} \left(\frac{1}{k^4 + \gamma^4} \right) = 1. \quad (61)$$

Thus the γ parameter is not independent, being defined as a function of the coupling constant g :

$$\gamma = \sqrt{2}\pi^2 N g^2. \quad (62)$$

It is clear that in close analogy to the Gribov-Zwanziger procedure [9, 10, 11] it is also desired to work directly with the gap equation $\frac{\delta\Gamma}{\delta\gamma^2} = 0$. It is well known that due to the fact that Γ corresponds to the energy in the euclidean case, $\varepsilon_v = 0$. It is necessary to break the supersymmetry explicitly in order to use this gap equation. The most simple way in order to do that is introducing a supersymmetry breaking term only which acts changing the value of the Gribov parameter in the fermionic partners

$$S_{break} = \rho \int d^3 x \bar{\lambda}^{a\alpha} \frac{\epsilon_{\alpha\beta}}{\partial^2} \lambda^{a\beta}. \quad (63)$$

This term breaks the supersymmetry and introduces, at principle, two different mass parameters that can be associated to a Gribov type behaviour. The original γ that appears in the gauge sector and the parameter ρ in the fermionic sector. Now, the equation $\frac{\delta\Gamma}{\delta\gamma^2} = 0$ is not trivial due to the fact that Γ is not zero anymore but a function of γ and ρ , $\Gamma(m, \rho, \gamma)$. Now the derivation in γ corresponds to a filtration only in the gauge sector, i.e. in the sector in which the γ appear. It is important to note that due to the structure of the Gribov propagator we have a gluino condensate given by $\lim_{x_1 \rightarrow x_2} Tr < \lambda_\alpha^a(x_1) \lambda_\beta^b(x_2) >$, which corresponds to the term added to the fermionic action. Thus the use of the gap equation permits to obtain the same integral condition as explained in the analysis of the ghost sector and used to obtain the value of γ . Again it is important to note that this procedure is equivalent to a filtration on gauge-ghost sector. At the end in order to recover the supersymmetry we impose $\Gamma(m, \rho(\gamma), \gamma) = 0$ and obtain again the original propagator for the gluino sector, fixing the ρ as the original value for this parameter that is obtained in the direct superfield formalism used in the construction of the Gribov term in section 3. Of course this method is much more adequate in the case of non explicit supersymmetric construction of the Gribov-Zwanziger procedure which is not the case in this work. The only advantage of the method of breaking and restoring the supersymmetry is to note that only the sector of the energy functional that is derived from the gauge fields contributes in order to obtain the value of γ . The results will be the same as in the more simple method explained in these section. The behaviour of the gauge and gaugino correlators is the same as presented in [31]. Of course due to the construction directly in superfields of our Gribov action it is not necessary the introduction, by hand, of the Gribov term for the fermionic sector, like in [31]. The price to pay for the explicitly supersymmetric construction is the need to look only to the ghost sector in order to fix the value of γ . In other terms, the explicitly superfield construction of the Gribov action gave us the prove that only the gauge-ghost sector is fundamental for the Gribov mechanism even in a supersymmetric action.

4 Some aspects of N= 1 Chern-Simons-Matter Theories

Now we will consider only the Chern-Simons sector. With interest in BLG and ABJM theories. Many works about BLG and ABJM exists in the literature like [38, 16, 39] and recently about BRST breaking in ABJM theory [40]. So we would focus on the possibility of a replica model [41] for confinement using the two Chern-Simons sectors of theories of that type. So considering only the Chern-Simons sector, we have that the ultraviolet mass dimension of gauge superfield Γ_α^a becomes $\frac{1}{2}$. In this case we rewrite the action (5) as

$$S_{SCS} = ik \int d^3 x d^2 \theta \left[(D^\alpha \Gamma^{a\beta})(D_\beta \Gamma_\alpha^a) + \frac{2}{3} i f^{abc} \Gamma^{a\alpha} \Gamma^{b\beta} (D_\beta \Gamma_\alpha^c) - \frac{1}{6} f^{abc} f^{cde} \Gamma^{a\alpha} \Gamma^{b\beta} \Gamma_\alpha^d \Gamma_\beta^e \right]. \quad (64)$$

As we are interested in N = 1 supersymmetric gauge field theory with the gauge group $G \times G$, we write a second action for another gauge superfield $\tilde{\Gamma}_\alpha^a$

$$\tilde{S}_{SCS} = i\tilde{k} \int d^3 x d^2 \theta \left[(D^\alpha \tilde{\Gamma}^{a\beta})(D_\beta \tilde{\Gamma}_\alpha^a) + \frac{2}{3} i f^{abc} \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}^{b\beta} (D_\beta \tilde{\Gamma}_\alpha^c) - \frac{1}{6} f^{abc} f^{cde} \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}^{b\beta} \tilde{\Gamma}_\alpha^d \tilde{\Gamma}_\beta^e \right]. \quad (65)$$

And we take the following total Chern-Simons action with matter:

$$S = S_{SCS} - \tilde{S}_{SCS} + S_{matter}. \quad (66)$$

With the matter action given by

$$S_{matter} = \int d^3x d^2\theta \text{tr} (\nabla^\alpha X^{I\dagger} \nabla_\alpha X_I + V), \quad (67)$$

with the matter superfield X in the bi-fundamental representation of the gauge group, i.e the superspace covariant derivatives for matrix-valued complex scalar superfields X^I and $X^{I\dagger}$ are defined by

$$\begin{aligned} \nabla_\alpha X^I &= D_\alpha X^I + i\Gamma_\alpha X^I - iX^I \tilde{\Gamma}_\alpha, \\ \nabla_\alpha X^{I\dagger} &= D_\alpha X^{I\dagger} - iX^{I\dagger} \Gamma_\alpha + i\tilde{\Gamma}_\alpha X^{I\dagger}, \end{aligned} \quad (68)$$

and V is the potential term given by

$$V = \frac{1}{k} \epsilon^{IJ} \epsilon_{KL} [X_I X^{K\dagger} X_J X^{L\dagger}]. \quad (69)$$

The classical Action (66) remains invariant under the following gauge transformation

$$\begin{aligned} \delta\Gamma_\alpha &= \nabla_\alpha \Lambda, & \delta\tilde{\Gamma}_\alpha &= \tilde{\nabla}_\alpha \tilde{\Lambda}, \\ \delta X^I &= i(\Lambda X^I - X^I \tilde{\Lambda}), & \delta X^{I\dagger} &= i(\tilde{\Lambda} X^{I\dagger} - X^{I\dagger} \Lambda), \end{aligned} \quad (70)$$

where $\Lambda = \Lambda^A T_A$ and $\tilde{\Lambda} = \tilde{\Lambda}^A \tilde{T}_A$ are parameters of transformations. The gauge invariance of this theory reflects that the theory have some spurious degrees of freedom. In order to quantize the theory correctly we need to fix the gauge. And we can do with the Faddeev-Popov method already studied, ending with an action of gauge fixing for each superfield Γ_α^a and $\tilde{\Gamma}_\alpha^a$, equation (11) [40].

In terms of the field components this action can represents the gauge part of the ABJM or BLG : with gauge group $G = SU(2)$, we can have a decomposition of BLG theory (It is possible to decompose the gauge symmetry generated by $SO(4)$ into $SU(2) \times SU(2)$) and with $G = U(N)$, we can have ABJM theory [38, 16, 39, 40]. In both cases we have, with $\tilde{k} = \pm k$, $G(N)_k \times G(N)_{\pm k}$, with k the Chern-Simons level.

We know that the solution of the Gribov problem for a Chern-Simons theory is trivial because the theory does not have metric [14]. But now, in this case with two Chern-Simons interacting with bi-fundamental matter, we can have a spontaneous symmetry breaking such that insert a metric [42, 43]. This open a possibility of seeing Gribov³ as a phase in ABJM theory, what we started to investigate here.

Here we only consider the two Chern-Simons actions in a toy model. Consider then the following mixing terms between the gauge fields.

$$S_{mix} = \mu \int d^3x d^2\theta \left(\Gamma^{a\alpha} \Gamma_\alpha^a - \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}_\alpha^a + 2\Gamma^{a\alpha} \tilde{\Gamma}_\alpha^a \right). \quad (71)$$

Where μ has the dimension of mass. We expect it is possible to get this mass and mixing term through spontaneous symmetry breaking [44]. And first we will address the following total action (levels k and $\tilde{k} = -k$)

$$S = S_{SCS(k)} - \tilde{S}_{SCS(k)} + S_{mix}. \quad (72)$$

So we get for full bilinear part of this total action

$$\begin{aligned} & \int d^3x d^2\theta \left[ik(D^\alpha \Gamma^{a\beta})(D_\beta \Gamma_\alpha^a) - ik(D^\alpha \tilde{\Gamma}^{a\beta})(D_\beta \tilde{\Gamma}_\alpha^a) + \mu \left(\Gamma^{a\alpha} \Gamma_\alpha^a - \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}_\alpha^a + 2\Gamma^{a\alpha} \tilde{\Gamma}_\alpha^a \right) \right] \\ &= \int d^3x d^2\theta \left[ik(\Gamma_\beta^a D^\alpha D^\beta \Gamma_\alpha^a) - ik(\tilde{\Gamma}_\beta^a D^\alpha D^\beta \tilde{\Gamma}_\alpha^a) + \mu \left(\varepsilon^{\alpha\beta} \Gamma_\beta^a \Gamma_\alpha^a - \varepsilon^{\alpha\beta} \tilde{\Gamma}_\beta^a \tilde{\Gamma}_\alpha^a + 2\varepsilon^{\alpha\beta} \Gamma_\beta^a \tilde{\Gamma}_\alpha^a \right) \right]. \end{aligned} \quad (73)$$

And we get the propagators in momentum space of the type of (49)

$$\begin{aligned} \langle \Gamma_\alpha^a(1) \Gamma_\beta^b(2) \rangle &= \frac{1}{8k^2} \delta^{ab} \left[\frac{(2kp^2 + \frac{2}{k}\mu^2)iD^2 + (p^2 + \frac{1}{2k^2}\mu^2)\mu}{(p^4 + \frac{1}{4k^4}\mu^4)} \right] \frac{D^2}{p^2} D_\beta D_\alpha \delta^2(\theta_1 - \theta_2) \delta^3(x_1 - x_2), \\ \langle \Gamma_\alpha^i(1) \Gamma_\beta^j(2) \rangle &= \langle \tilde{\Gamma}_\alpha^i(1) \tilde{\Gamma}_\beta^j(2) \rangle, \end{aligned} \quad (74)$$

³Gribov confinement scenario at level of propagator, namely, if we can get a Gribov type propagator.

and a mixing propagator

$$\langle \Gamma_\alpha^a(1) \tilde{\Gamma}_\beta^b(2) \rangle = -\frac{\mu}{8k^2} \delta^{ab} \left[\frac{p^2 - \frac{1}{2k^2} \mu^2 - \frac{1}{k} i \mu D^2}{(p^4 + \frac{1}{4k^4} \mu^4)} \right] \frac{D^2}{p^2} D_\beta D_\alpha \delta^2(\theta_1 - \theta_2) \delta^3(x_1 - x_2). \quad (75)$$

In the strong coupling regime (taking $k = 1$) this propagators have the pole of Gribov type.

Let's address now the action in a regime with levels k and $\tilde{k} = k$:

$$S = S_{SCS_{(k)}} + \tilde{S}_{SCS_{(k)}} + S_{mix}. \quad (76)$$

We get for full bilinear part of this total action

$$\begin{aligned} & \int d^3x d^2\theta \left[ik(D^\alpha \Gamma^{a\beta})(D_\beta \Gamma_\alpha^a) + ik(D^\alpha \tilde{\Gamma}^{a\beta})(D_\beta \tilde{\Gamma}_\alpha^a) + \mu \left(\Gamma^{a\alpha} \Gamma_\alpha^a - \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}_\alpha^a + 2\Gamma^{a\alpha} \tilde{\Gamma}_\alpha^a \right) \right] \\ & = \int d^3x d^2\theta \left[ik(\Gamma_\beta^a D^\alpha D^\beta \Gamma_\alpha^a) + ik(\tilde{\Gamma}_\beta^a D^\alpha D^\beta \tilde{\Gamma}_\alpha^a) + \mu \left(\varepsilon^{\alpha\beta} \Gamma_\beta^a \Gamma_\alpha^a - \varepsilon^{\alpha\beta} \tilde{\Gamma}_\beta^a \tilde{\Gamma}_\alpha^a + 2\varepsilon^{\alpha\beta} \Gamma_\beta^a \tilde{\Gamma}_\alpha^a \right) \right]. \end{aligned} \quad (77)$$

And we get the propagators in momentum space:

$$\begin{aligned} \langle \Gamma_\alpha^a(1) \Gamma_\beta^b(2) \rangle &= \frac{1}{8k^2} \delta^{ab} \left[\frac{(2kp^2 - \frac{1}{k} \mu^2) i D^2 + (p^2 + \frac{1}{2k^2} \mu^2) \mu}{(p^2 + \frac{1}{2k^2} \mu^2)^2} \right] \frac{D^2}{p^2} D_\beta D_\alpha \delta^2(\theta_1 - \theta_2) \delta^3(x_1 - x_2), \\ \langle \Gamma_\alpha^i(1) \Gamma_\beta^j(2) \rangle &= \langle \tilde{\Gamma}_\alpha^i(1) \tilde{\Gamma}_\beta^j(2) \rangle, \end{aligned} \quad (78)$$

and a mixing propagator

$$\langle \Gamma_\alpha^a(1) \tilde{\Gamma}_\beta^b(2) \rangle = -\frac{\mu}{8k^2} \delta^{ab} \left[\frac{1}{p^2 + \frac{1}{2k^2} \mu^2} \right] \frac{D^2}{p^2} D_\beta D_\alpha \delta^2(\theta_1 - \theta_2) \delta^3(x_1 - x_2). \quad (79)$$

This regime is responsible for massive particles. So we obtain in this toy model at least two phases, one of which is confinement in Gribov-Zwanziger scenario.

5 Conclusions

In this work we have studied the Super-Yang-Mills Chern-Simons theory, $N = 1$, considering the Gribov problem present in YM theories as well as a generalization of the Gribov-Zwanziger approach with auxiliary superfields in superspace. A local supersymmetric Gribov-Zwanziger sector is presented providing the starting point in order to implement the restriction to the first Gribov region beyond one-loop order. Which results in propagators of the Gribov type opening a perspective of treating confinement in these theories in terms of Gribov-Zwanziger scenario. And we note that Gribov acts as a regulator for the usual infrared SYM-CS theory, which can be observed from super propagator (49) Also a possible mechanism in order to obtain a Gribov like propagator from a ABJM theory is presented. It suggests that it is possible by making use of a spontaneous symmetry breaking mechanism similar to the one proposed in [45], which is under investigation. This mechanism can offer a possibility of seeing Gribov as a phase in ABJM theory, offering a link between confinement in Gribov-Zwanziger scenario and confinement as seeing by the brane scenario. We expect to study the implications of this relation in future works, specially the possibility of obtaining a condensate that can be associated to a usual particle in Källen-Lehmann representation.

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A Notation, conventions and some useful formulas

We work with Euclidean metric: $\text{diag}(+++)$. So we choose the gamma matrices being the Pauli matrices σ_i [[46]]:

$$\gamma^\mu \equiv (\sigma_\mu)_\alpha^\beta \quad (80)$$

which are OS self-conjugate and:

$$\{\sigma^\mu, \sigma^\nu\} = 2\delta^{\mu\nu} I, \quad (81)$$

$$[\sigma^\mu, \sigma^\nu] = 2i\varepsilon^{\mu\nu\sigma} \sigma^\sigma. \quad (82)$$

The invariant anti-symmetric tensor is defined as

$$\varepsilon^{-+} = \varepsilon_{-+} = +1, \quad (83)$$

$$\varepsilon^{\gamma\beta} \varepsilon_{\beta\alpha} = -\delta_{\alpha}^{\gamma}, \quad (84)$$

and are used to raise and lower indices as conversion:

$$\psi^{\alpha} = \varepsilon^{\alpha\beta} \psi_{\beta}, \quad (85)$$

$$\psi_{\alpha} = \psi^{\beta} \varepsilon_{\beta\alpha}. \quad (86)$$

In this way is possible to find the representation of differential operator of the generators of super algebra in D=3, with the concept of graded Majorana [46]:

$$Q_{\alpha} = -\partial_{\alpha} + \partial_{\alpha\beta} \theta^{\beta}, \quad (87)$$

with

$$\partial_{\alpha\beta} = i\sigma_{\alpha}^{\mu\gamma} \varepsilon_{\gamma\beta} \partial_{\mu}. \quad (88)$$

As well as the superspace derived:

$$D_{\alpha} = \partial_{\alpha} + \partial_{\alpha\beta} \theta^{\beta}, \quad (89)$$

with the following relations:

$$\{D_{\alpha}, D_{\beta}\} = 2\partial_{\alpha\beta}, \quad (90)$$

$$[D_{\alpha}, D_{\beta}] = -2\varepsilon_{\alpha\beta} D^2, \quad (91)$$

$$D_{\alpha} D_{\beta} = \partial_{\alpha\beta} - \varepsilon_{\alpha\beta} D^2, \quad (92)$$

$$D^{\beta} D_{\alpha} D_{\beta} = 0. \quad (93)$$

And it is easy to verify that

$$[Q_{\alpha}, D_{\beta}] = 0. \quad (94)$$

Another useful relations:

$$\partial_{\alpha\beta} \partial^{\alpha\gamma} = \partial^2 \delta_{\beta}^{\gamma}, \quad (95)$$

$$(D^2)^2 = -\partial^2, \quad (96)$$

$$\int d^2\theta = -\frac{1}{4} D^2, \quad (97)$$